

1] Capacitors

symbol

two plates charged with ac source

$8.85 \times 10^{-12} \text{ F/m}$   
 $\epsilon_0 \epsilon_r A$  permittivity  
 $d$  dist. betw. plates  
 area of plates

Capacitance =  $C = \frac{Q \rightarrow \text{charge}}{V \rightarrow \text{applied voltage}}$   
 or  $Q = CV$

its unit is Farad (F)  $\rightarrow$  actually (micro, nano, Pico Farad)

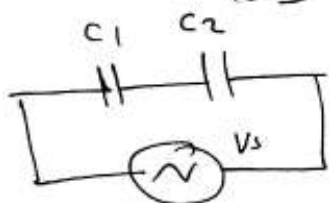
EX(1) A certain capacitor stores 50 microcoulombs with 10V across its plates.  
 What is capacitance (in microfarad)

sol/  $C = Q/V = \frac{50 \text{ micro}}{10} = 5 \mu\text{F}$

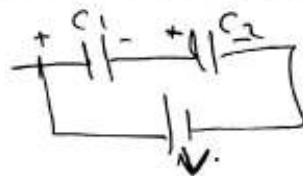
EX(2) determine the capacitance of parallel plates having area  $0.01 \text{ m}^2$   
 & separation  $0.5 \text{ mil} = [1.27 \times 10^{-5} \text{ m}]$ , Dielectric is mica (has  $\epsilon_r$  of 5)

sol  $C = \frac{\epsilon_0 \epsilon_r A}{d} = \frac{(8.85 \times 10^{-12})(5)(0.01)}{1.27 \times 10^{-5}} = 0.035 \mu\text{F}$

a- Series connection of capacitors



or



note  $Q_1 = Q_2 = Q_3 = \dots$   
 $\therefore C_T = \frac{C}{n}$   
 (تقسیم، عدد)

$V_T = V_1 + V_2 + \dots$

$\frac{Q_T}{C_T} = \frac{Q_1}{C_1} + \frac{Q_2}{C_2} + \dots$

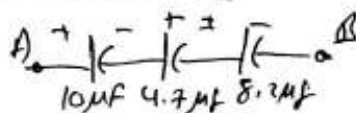
$\therefore \frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} + \dots$

$Q_1 = Q_2 = Q_3 = \dots$

series = (تسلسلی) (مجموعه)  
 Parallel = (متوازی) (مجموعه)

Series Capacitor Like Parallel Resistors

EX(3) Find total capacitance between A & B



$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \quad \therefore \frac{1}{C_T} = \frac{1}{2.3 \mu\text{F}} \quad \therefore C_T = 2.3 \mu\text{F}$

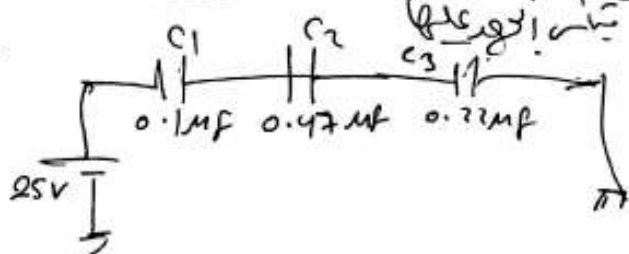
(2)

note if you want to use voltage divider in series connection to determine voltage across each individual capacitor, So:-

$$V_1 = V_T \times \frac{X_{C1}}{X_{CT}} = \left( V_T \cdot \frac{C_T}{C_1} \right) \rightarrow \text{total cap}$$

EX(4) find voltage across each capacitor

$$C_T = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}} = 0.06 \mu\text{F}$$



$$V_1 = V_T \frac{C_T}{C_1} = 25 \times \frac{0.06 \mu\text{F}}{0.1 \mu\text{F}} = 15\text{V}$$

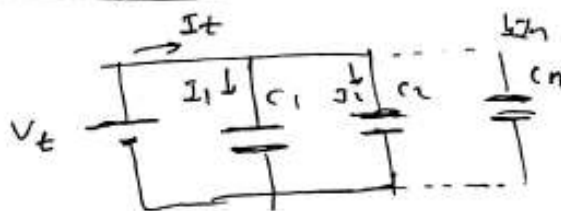
$$V_2 = V_T \frac{C_T}{C_2} = 25 \times \frac{0.06 \mu\text{F}}{0.47 \mu\text{F}} = 3.19\text{V}$$

$$V_3 = V_T \frac{C_T}{C_3} = 25 \times \frac{0.06 \mu\text{F}}{0.22 \mu\text{F}} = 6.81\text{V}$$

### b - Parallel Connection of C

$$Q_T = Q_1 + Q_2 + Q_3 + \dots + Q_n$$

$$C_T V_T = C_1 V_1 + C_2 V_2 + C_3 V_3 + \dots + C_n V_n$$

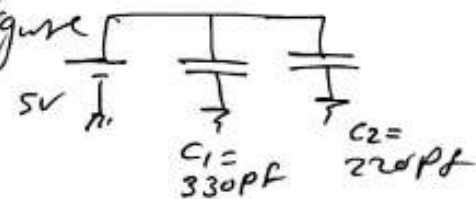


but  $V_T = V_1 = V_2 = \dots = V_n$

$$\therefore C_T = C_1 + C_2 + C_3 + \dots + C_n$$

So parallel  $C \equiv$  series  $R$

EX(5) what is the voltage across each cap of figure

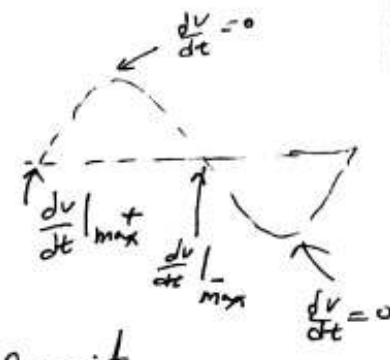
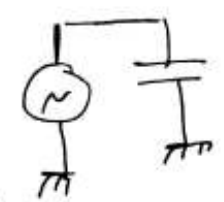


$$C_T = C_1 + C_2 = 330 + 220 = 550 \text{ pF}$$

$$V_T = V_1 = V_2 = 5\text{V}$$

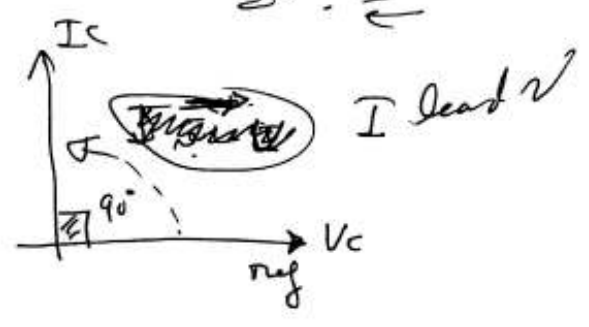
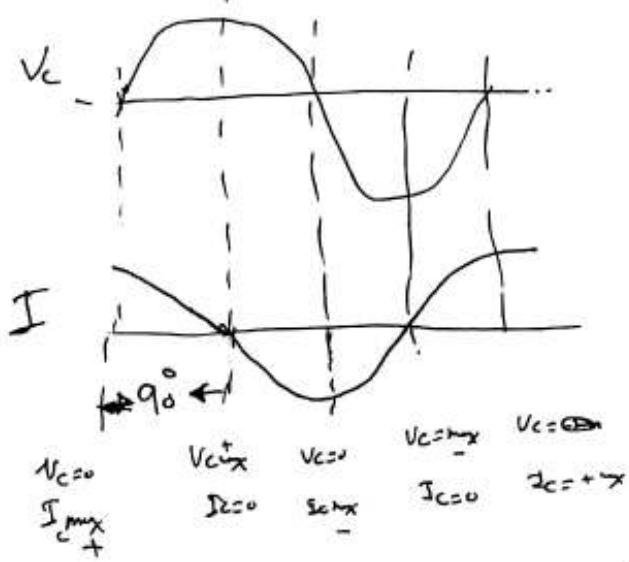
(3)

in capacitor  
 $i = c \frac{dv}{dt}$



Relation between I & V in capacitor

**I** lead **V** by  $90^\circ$  in pure capacitive circuit



$\Sigma \text{ new } V$

also written  
 $-jX_c$  or  $\frac{1}{jX_c}$   
 phase  $(90^\circ)$

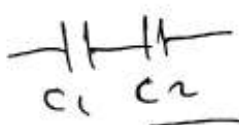
Capacitive Reactance

$X_c = \frac{1}{2\pi f c}$   
 magnitude

Ex(6) For the circuit shown, determine capacitive reactance of  $f = 1 \text{ kHz}$

Sol:  $X_c = \frac{1}{2\pi f c} = \frac{1}{2\pi \times 10^3 \times 0.0047 \times 10^{-6}}$  Vs  $= 37.9 \text{ k}\Omega$

(a)  $X_{CT} = X_{C1} + X_{C2}$

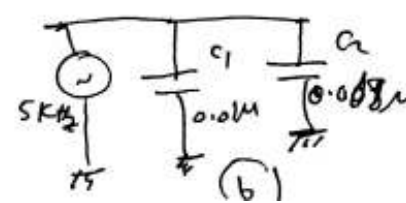
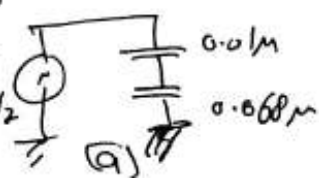


$X_{CT} = X_{C1} + X_{C2}$

$X_{CT} = \frac{X_{C1} \cdot X_{C2}}{X_{C1} + X_{C2}}$

Ex(7) Calc. total capacitive reactance for  $\odot$ 's

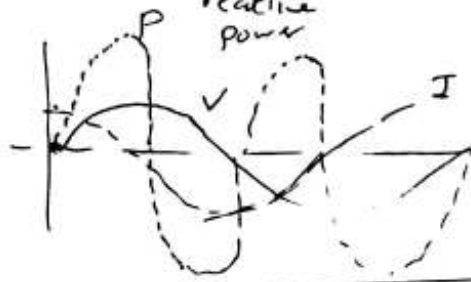
(a)  $X_{CT} = X_{C1} + X_{C2} = \frac{1}{2\pi \times 5 \text{ kHz} \times 0.01 \mu} + \frac{1}{2\pi \times 5 \text{ kHz} \times 0.008 \mu}$   
 $= 408 \mu$



(4)

Power in capacitor

$$P = V_{rms} I_{rms} = \frac{V_{rms}^2}{X_c} = I_{rms}^2 X_c$$



→ True Power = 0 (ideal capacitor)  
يعني

Note:- Power supply filters also capacitor  
يعني

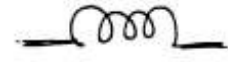
## 2- Inductors (coils)

coil with (N) turns

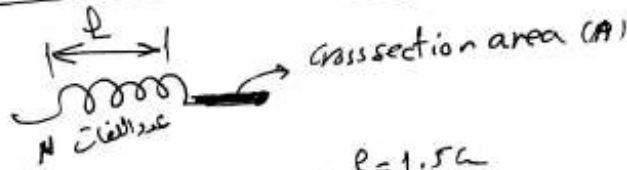
the induced voltage  $(V_{ind} = L \frac{di}{dt})$

inductance (henry)

current rate (Amp/sec)

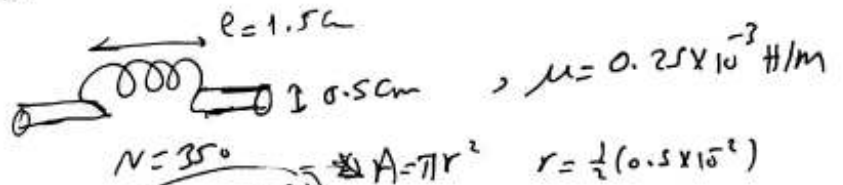


$$L = \frac{N^2 \mu A}{l}$$



يعني  $A = \pi r^2$   
مساحة القطر

$\Sigma X$

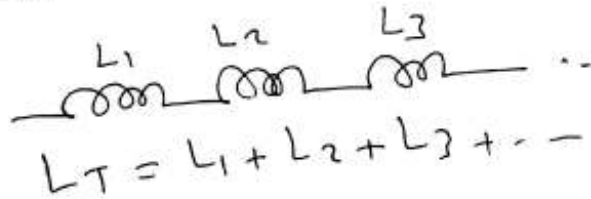


$\mu = 0.25 \times 10^{-3} \text{ H/m}$

$A = \pi r^2$   $r = \frac{1}{2} (0.5 \times 10^{-2})$

$$L = \frac{(350)^2 \times (0.25 \times 10^{-3}) \times (\pi (0.25 \times 10^{-2})^2)}{1.5 \times 10^{-2}} = 40 \text{ mH}$$

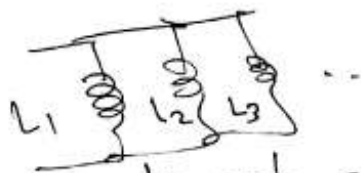
Series connection



$$L_T = L_1 + L_2 + L_3 + \dots$$

سلسلة (سلسلة)

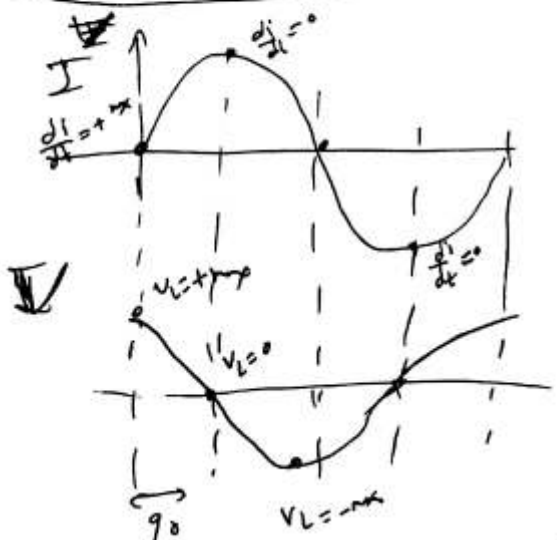
Parallel connection



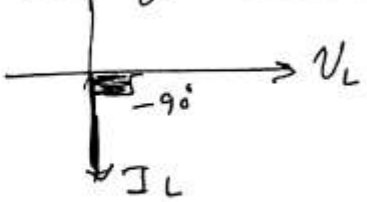
$$\frac{1}{L_T} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \dots$$

متوازية (متوازية)

Relation between I & V in inductor



I lag V by 90°

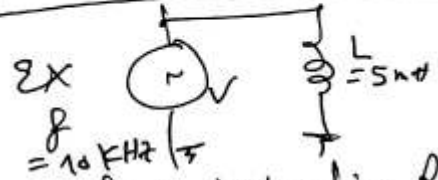


inductive reactance

$$X_L = 2\pi fL$$

مقاومت القابض، الخرجه

$$jX_L = j2\pi fL = j\omega L$$



find inductive reactance

Sol:  $X_L = 2\pi fL = 2\pi \times 10 \times 10^3 \times (5 \times 10^{-3}) = 314 \Omega$

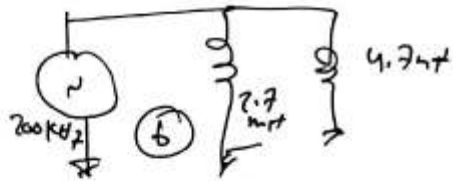
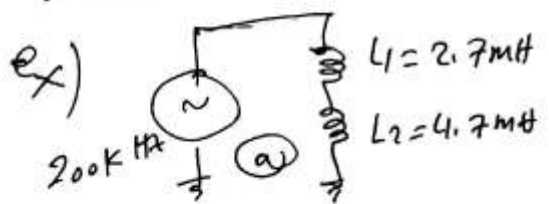


$$X_{LT} = X_{L1} + X_{L2} + \dots$$



$$\frac{1}{X_{LT}} = \frac{1}{X_{L1}} + \frac{1}{X_{L2}} + \dots$$

مقاومت القابض  
مقاومت القابض  
R و jX\_L



$$X_{L1} = 2\pi fL_1 = 2\pi \times (200 \times 10^3) \times (2.7 \times 10^{-3}) = 3.39 \text{ k}\Omega$$

$$X_{L2} = 2\pi fL_2 = 2\pi \times (200 \times 10^3) \times (4.7 \times 10^{-3}) = 5.91 \text{ k}\Omega$$

fig (a)  $X_{L \text{ tot}} = X_{L1} + X_{L2} = 9.3 \text{ k}\Omega$

fig (b)  $X_{L \text{ tot}} = \frac{X_{L1} \cdot X_{L2}}{X_{L1} + X_{L2}} = 2.15 \text{ k}\Omega$

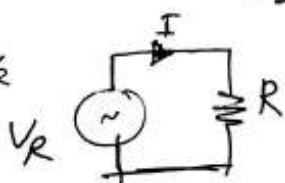
Power in inductor

$$P_{\text{react}} = V_{\text{rms}} I_{\text{rms}} = \frac{V_{\text{rms}}^2}{X_L} = I_{\text{rms}}^2 X_L$$

# ⑥ R, L, C in Circuits

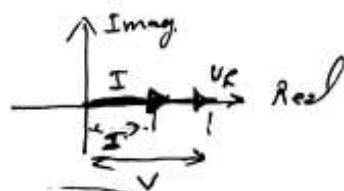
- ① Pure Resistance    ② Pure capacitance    ③ Pure inductance  
 ④ series RL    ⑤ Series RC    ⑥ series RLC  
 ⑦ General Case    ⑧ Problem    ⑨ Admittance

## ① Pure Resistance

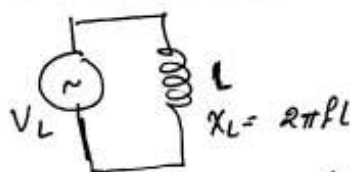
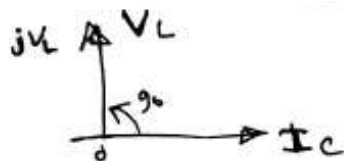


impedance  $Z = \frac{V_R}{I} = \frac{V_R \angle 0^\circ}{I_R \angle 0^\circ} = R \angle 0^\circ$

note:  $V, I$  in the same phase in pure resistance



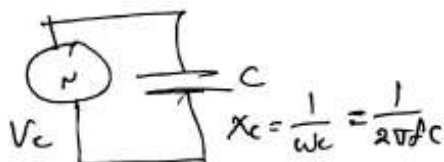
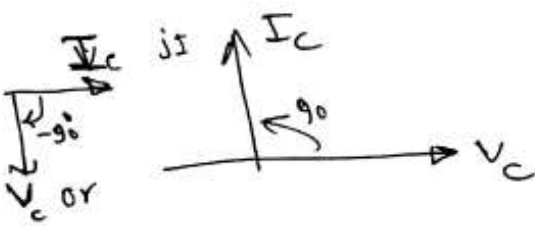
## ② Pure inductance



$Z = \frac{V_L \angle 90^\circ}{I_L \angle 0^\circ} = X_L \angle 90^\circ = jX_L = j\omega L = j2\pi fL$

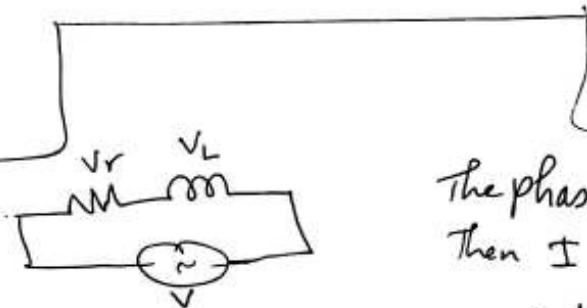
$I \text{ lag } V$

## ③ Pure capacitance

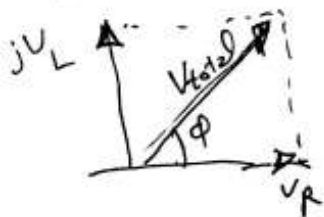


$Z = \frac{V_C \angle 0^\circ}{I_C \angle 90^\circ} = X_C \angle -90^\circ = -jX_C = -\frac{j}{\omega C} = \frac{-j}{2\pi fC}$

## ④ series RL



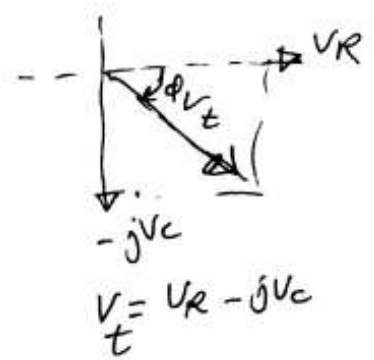
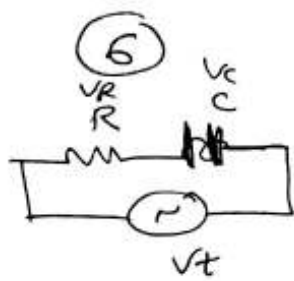
The phasor is sum of  $V_R$  &  $V_L$   
 Then  $I$  lag  $V$  by angle between  $(0, 90^\circ)$



$Z = 3 + j4 = 5 \angle \tan^{-1} \frac{4}{3}$   
 (where 3 is R, 4 is  $X_L = 2\pi fL$ , and 5 is magnitude)

5 - series RC

I lead V by angle between (0 to 90°)



$$Z = R - jX_c = R - j\left(\frac{1}{\omega C}\right)$$

$$= \left[ R + X_c = R + \frac{1}{j\omega C} = R - j\left(\frac{1}{\omega C}\right) \right]$$

magnitude =  $\sqrt{R^2 + X_c^2}$  , phase  $\tan^{-1}\left(-\frac{X_c}{R}\right)$

6 - Series RLC

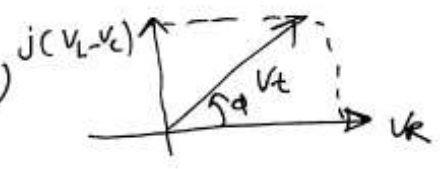


$$V = V_R + j(V_L - V_C)$$

$$Z = R + j(X_L - X_C)$$

$$= \sqrt{R^2 + (X_L - X_C)^2} \left[ \tan^{-1}\left(\frac{X_L - X_C}{R}\right) \right]$$

note This circuit used in Resonance circuit

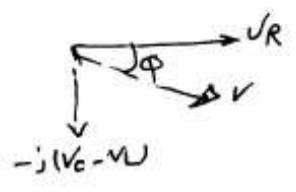


There are 3 cases

1- if  $X_L = X_C$  ( $V_L = V_C$ )  $\therefore Z = R$  (pure resistance)  
 (resonance)  $\omega L = \frac{1}{\omega C}$

2- if  $X_L > X_C$  ( $V_L > V_C$ )  $\therefore Z = R + j(X_L - X_C)$   
 circuit is inductive effect

3- if  $X_L < X_C$  ( $V_L < V_C$ )  $\therefore Z = R + j(X_L - X_C)$   
 circuit is capacitive effect



(7)  
Parallel AC

Admittance  $Y = \frac{1}{Z}$  (Siemen, S)

$Y = G + jB$  → susceptance (imaginary part of admittance)  
 ↳ Conductance (real of admittance)

1- For pure Resistance  $Z = R$  or  $Y = \frac{1}{Z} = \frac{1}{R} = G$

2- For pure inductance  $Z = jX_L$  &  $Y = \frac{1}{jX_L} = \frac{-j}{X_L} = -jB_L$   
inductive susceptance

3- For pure capacitance  $Z = -jX_C \Rightarrow Y = \frac{1}{Z} = \frac{1}{-jX_C} = \frac{j}{X_C} = +jB_C$

4- Series RL  $Z = R + jX_L \Rightarrow Y = \frac{1}{R + jX_L} = \frac{R - jX_L}{R^2 + X_L^2}$   
 $= \frac{R}{R^2 + X_L^2} - j \frac{X_L}{R^2 + X_L^2} \rightarrow B = -\frac{X_L}{Z^2}$   
↳  $G = \frac{R}{Z^2}$

5- Series RC  $Z = R - jX_C$  &  $Y = \frac{1}{R - jX_C} = \frac{R + jX_C}{R^2 + X_C^2}$   
 $= \frac{R}{R^2 + X_C^2} + j \frac{X_C}{R^2 + X_C^2} \rightarrow B = \frac{X_C}{Z^2}$   
↳  $G = \frac{R}{Z^2}$

6- RL parallel  $\frac{1}{Z} = \frac{1}{R} + \frac{1}{jX_L}$

$Y = \frac{1}{Z} = \frac{1}{R} - j \frac{1}{X_L} = \frac{1}{R} - j \frac{1}{X_C} \rightarrow B = -\frac{1}{X_C}$   
↳  $G = \frac{1}{R}$

7- RC Parallel  $\frac{1}{Z} = \frac{1}{R} + \frac{1}{-jX_C}$   
 $Y = \frac{1}{Z} = \frac{1}{R} + \frac{j}{X_C} \rightarrow B$



Problems

(Slide 66 → 90)  
≡ Sheet (1)

EX (\*) -

determine the values of the resistance & series connected inductance or capacitance for each of the following impedances:-  
(a)  $12 + j5 \Omega$       (b)  $-j40 \Omega$       (c)  $30 \angle 60^\circ$       (d)  $2.2 \times 10^6 \angle -30^\circ \Omega$

If the freq. of the source is  $50 \text{ Hz}$

Solve ~~with~~ ~~the~~ ~~following~~ ~~steps~~ ~~and~~ ~~find~~ ~~the~~ ~~values~~ ~~of~~ ~~R~~ ~~and~~ ~~L~~ ~~or~~ ~~C~~

Series R & L  
∴  $R = 12 \Omega$

(a)  $12 + j5 \equiv R + jX_L$

$X_L = 5 = 2\pi fL = 2\pi \times 50 L \Rightarrow L = 0.0159 \text{ H}$

(b)  $-j40 = \text{Pure Capacitance only}$        $X_C = 40 = \frac{1}{2\pi fC} \Rightarrow C = 79.6 \text{ nF}$

(c)  $30 \angle 60^\circ = 30 [\cos 60^\circ + j \sin 60^\circ] = 15 + j25.98$

= series R, L       $\Rightarrow R = 15 \Omega$        $X_L = 25.98 = 2\pi fL$   
 $L = 0.0827 \text{ H}$

(d)  $2.2 \times 10^6 \angle -30^\circ = 2.2 \times 10^6 [\cos(-30^\circ) + j \sin(-30^\circ)] = 1.905 \times 10^6 - j1.1 \times 10^6$

series R & C      ∴  $R = 1.905 \text{ M}\Omega$   
 $X_C = 1.1 \times 10^6 = \frac{1}{2\pi fC} \Rightarrow C = 2.89 \times 10^{-9} \text{ F}$   
 $= 2.89 \text{ nF}$

9

EX(2) :- determine in Polar & rectangular forms the current flowing in inductor of negligible resistance & inductance  $159.2\text{mH}$  when it is connected to  $250\text{V}$ ,  $50\text{Hz}$  supply



inductive

$$X_L = 2\pi fL = 2\pi(50)(159.2 \times 10^{-3}) = 50\Omega$$

$$Z = R + jX_L = 0 + j50 = 50 \angle 90^\circ \Omega$$

$$V = 250 \angle 0^\circ = 250 + j0$$

$$I = \frac{V}{Z} = \frac{250 \angle 0^\circ}{50 \angle 90^\circ} = 5 \angle -90^\circ \text{ A}$$

$A + jB$  Polar form  
 $\sqrt{A^2 + B^2} \angle \tan^{-1} \frac{B}{A}$  Rectangular form

EX(3) A  $200\text{V}$ ,  $50\text{Hz}$  supply is connected across a coil with inductance  $0.15\text{H}$  & has a series resistance of  $32\Omega$ , determine

→ impedance of the circuit

→ current & phase

→ P.d. across  $32\Omega$

→ " " " " coil

sol

$$\text{a) } X_L = 2\pi fL = 2\pi(50)(0.15) = 47.1\Omega$$

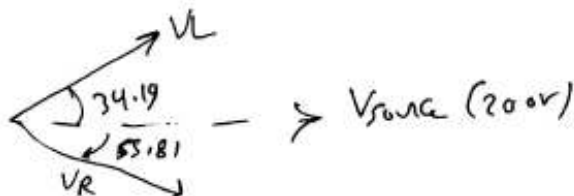
$$Z = R + jX_L = 32 + j47.1 = 57 \angle 55.81^\circ$$

$$\text{b) } I = \frac{V}{Z} = \frac{200 \angle 0^\circ}{57 \angle 55.81^\circ} = 3.51 \angle -55.81^\circ$$

$$\text{c) P.d. in } 32\Omega \quad V_R = I R = 3.51 \angle -55.81^\circ \times (32)$$

$$V_R = (3.51 \angle -55.81^\circ) (32 \angle 0^\circ) = 112.3 \angle -55.81^\circ$$

$$\text{d) P.d. across coil} = I X_L = 3.51 \angle -55.81^\circ \times 47.1 \angle 90^\circ = 165.3 \angle 34.19^\circ$$



(10)

parallel

EX(4) The admittance of a circuit is  $(0.04 + j0.025)S$ , determine

The value of R R C if they are connected  $\leftarrow$  Parallel series

2. Draw diagram  $\frac{1}{f} = 50Hz$

a-Parallel

$$Y = \underbrace{0.04}_{\rightarrow G} + j \underbrace{0.025}_{\rightarrow B} = \frac{1}{R} + j\left(\frac{1}{X_c}\right)$$



$$\frac{1}{R} = 0.04 \quad \therefore R = \frac{1}{0.04} = \underline{25\Omega}$$

$$\frac{1}{X_c} = 0.025 \quad \text{or } X_c = 40\Omega = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50 C}$$

$$\therefore C = \underline{79.5\mu F}$$

b-Series

$$Y = 0.04 + j0.025 = \frac{1}{Z}$$
$$Z = \frac{1}{0.04 + j0.025} \times \frac{0.04 - j0.025}{0.04 - j0.025} = \frac{0.04 - j0.025}{(0.04)^2 + (0.025)^2}$$
$$= \underline{17.98 - j11.24} = R - jX_c$$

$$\text{where } R = \underline{17.98\Omega}$$

$$X_c = \underline{11.24\Omega}$$

11

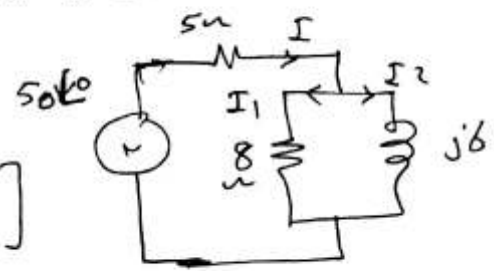
EX(5) Determine  $I, I_1, I_2$  in shown circuit

$$Z_t = 5 + 8 // j6 = 5 + \frac{(8)(j6)}{8+j6}$$

$$= 5 + \frac{48j}{8+j6} = 5 + \left[ \frac{48j}{8+j6} \cdot \frac{8-j6}{8-j6} \right]$$

$$= 5 + \frac{348j + 288}{(8)^2 + (6)^2 = 100} = 5 + 3.84j + 2.88 = \underline{7.88 + 3.84j}$$

$$= 8.77 \angle 25.98^\circ$$



$$I = \frac{V}{Z} = \frac{50 \angle 0^\circ}{8.77 \angle 25.98^\circ} = 5.7 \angle -25.98^\circ \text{ A}$$

$$I_1 = I \times \frac{j6}{8+j6} \quad \text{Current divider} = (5.7) \angle -25.98^\circ \times \frac{6 \angle 90^\circ}{10 \angle 36.87^\circ}$$

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$$I_1 = \frac{5.7 \times 6}{10} \angle -25.98 + 90 - 36.87 = 3.42 \angle 27.15^\circ \text{ A}$$

$$I_2 = I \left( \frac{8}{8+j6} \right) = 5.7 \angle -25.98^\circ \times \frac{8 \angle 0^\circ}{10 \angle 36.87^\circ}$$

$$= 4.56 \angle -62.85^\circ \text{ A}$$

EX(6) For shown circuit determine  $I$  & phase

$Z_1 = 5 + j12$        $Z_2 = 3 - j4$        $Z_3 = 8$

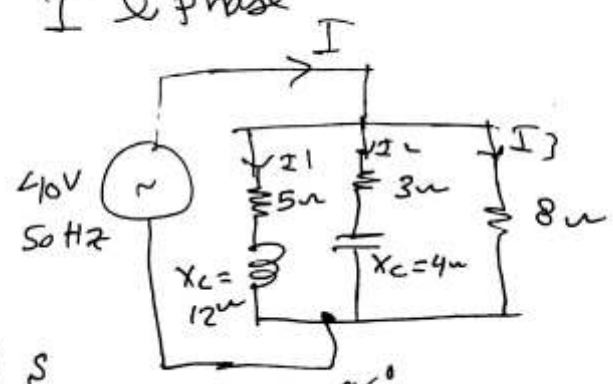
$Y_t = Y_1 + Y_2 + Y_3$  (Parallel admittance)

$$= \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3} = \frac{1}{5+j12} + \frac{1}{3-j4} + \frac{1}{8}$$

$$Y_t = \frac{5-j12}{5^2+12^2} + \frac{3-j4}{3^2+4^2} + \frac{1}{8} = (0.2746 + j0.089) \text{ S} = 0.2887 \angle 17.96^\circ$$

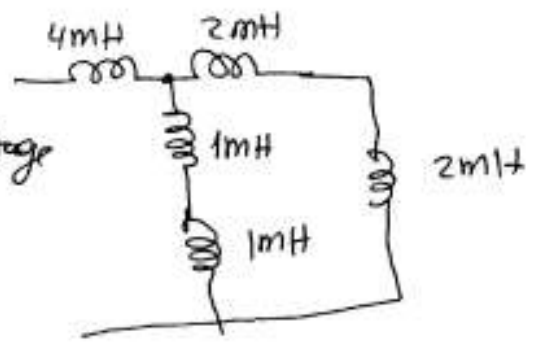
$$I = \frac{V}{Z} = V Y = 40 \angle 0^\circ \times 0.2887 \angle 17.96^\circ = 11.548 \angle 17.96^\circ$$

$\therefore I$  lead supply voltage by  $17.96^\circ$



سوال ۱۸ (۱۲)

Q) what frequency will produce a total current (rms) of 500 mA if voltage = 10V



$$\text{Sol } Z = \frac{V}{I} = \frac{10}{500 \text{ mA}} = 20 = X_{\text{leg}}$$

$$Z_{\text{eq}} = (4 \text{ mH}) + (2 \text{ mH} \parallel 2 \text{ mH}) = 5.33 \text{ mH}$$

$$X_{\text{leg}} = 2\pi f \times 5.33 \text{ mH} = 20$$

$$\therefore f = \frac{20}{2\pi \times 5.33 \times 10^{-3}} = 596.8 \text{ Hz}$$

~~Q1~~